(CHAPTER 3)

Fluid Statics

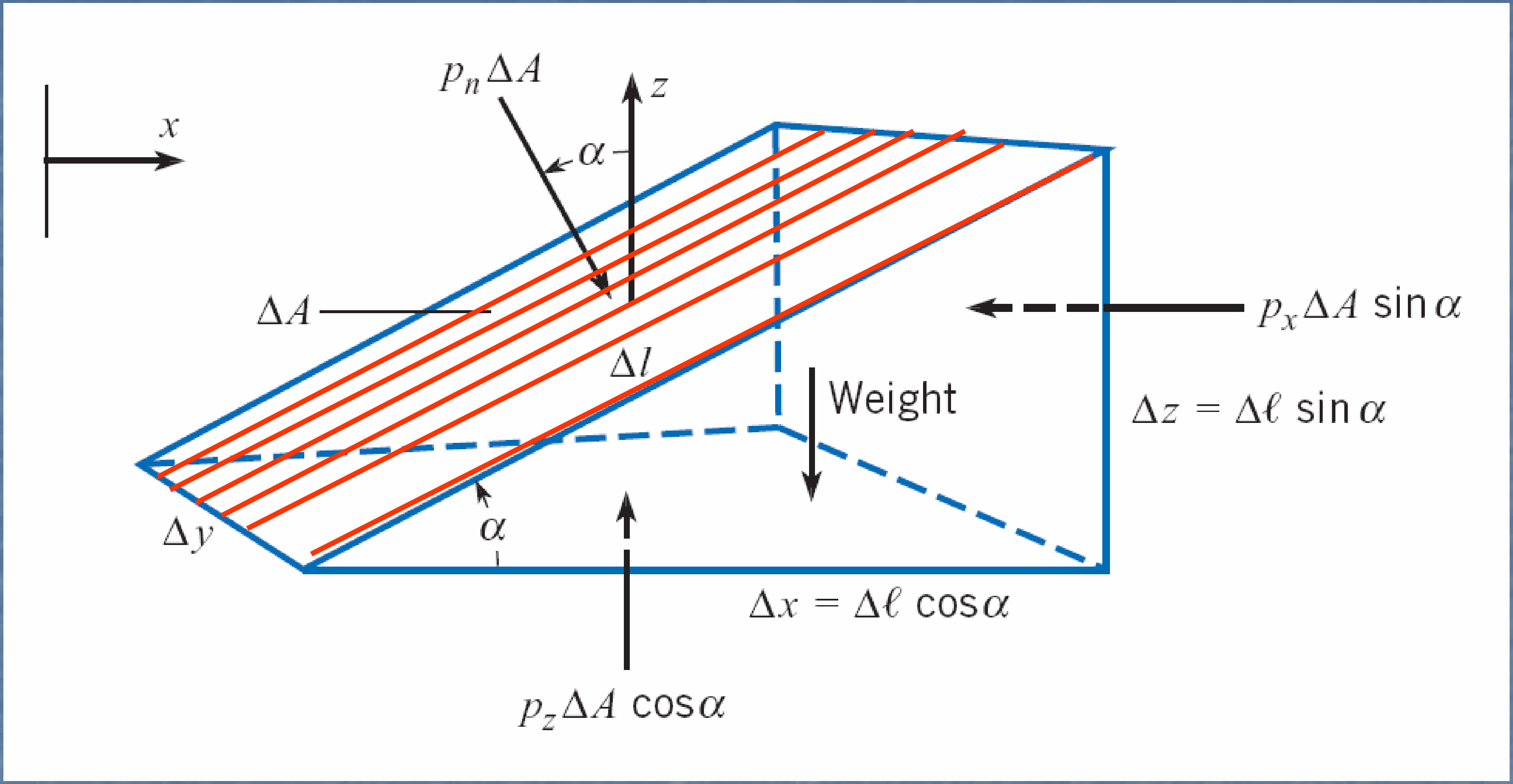
1. PRESSURE

For a static Fluid, hydrostatic pressure by definition

$$P = \frac{F}{A}$$

To prove this, consider a wedge-shaped element of fluid in equilibrium as shown in Fig. 1

Fluid Statics



Consider the forces in the X-direction, we obtain

Fluid Statics

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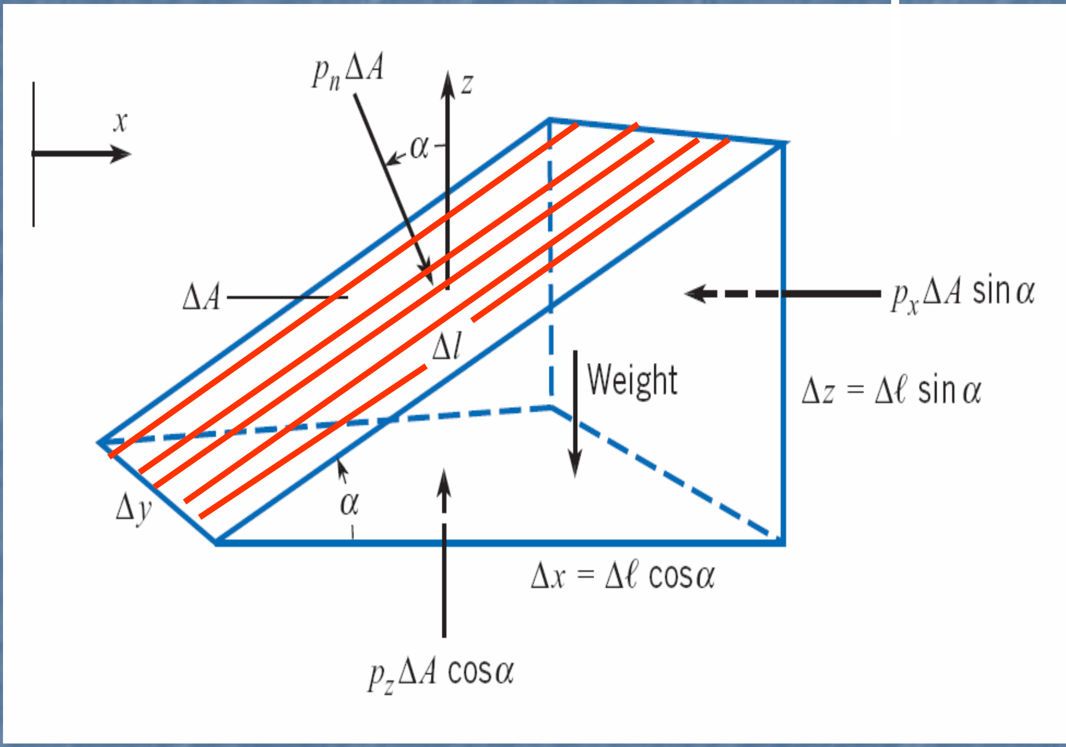
+Ve (Z)

$$(p_n \Delta y \Delta l)\sin\alpha - p_x(\Delta y \Delta l \sin\alpha) = 0$$

Consider the forces in the Z-direction, we obtain



$$-(p_n \Delta y \Delta l)\cos\alpha + p_z(\Delta y \Delta l \cos\alpha) - mg = 0$$

$$-(p_n \Delta y \Delta l)\cos\alpha + p_z(\Delta y \Delta l \cos\alpha) - \rho V g = 0$$

Note: Volume of a triangle (V)

= Area X height = $\frac{1}{2}(\Delta y \Delta l \cos\alpha)\Delta z)$

$$\frac{1}{2}(\Delta y \Delta l \cos\alpha)(\Delta l \sin\alpha)$$

$$-(p_n \Delta y \Delta l)\cos\alpha + p_z(\Delta y \Delta l \cos\alpha) - \rho g\left(\frac{1}{2}(\Delta y \Delta l \cos\alpha)(\Delta l \sin\alpha)\right) = 0$$

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Dividing by the term $(\Delta y \Delta l \cos\alpha)$ and let $(\Delta l \rightarrow 0)$

$p_n = p_z = 0$ and $p_n = p_x = p_z = 0$

Since angle (α) is arbitrary and pressure (p_n) is independent of (α), then it can be concluded that the pressure at a point in a static fluid acts with the same magnitude in all directions.

$$p_n = p_x = p_y = p_z = 0$$

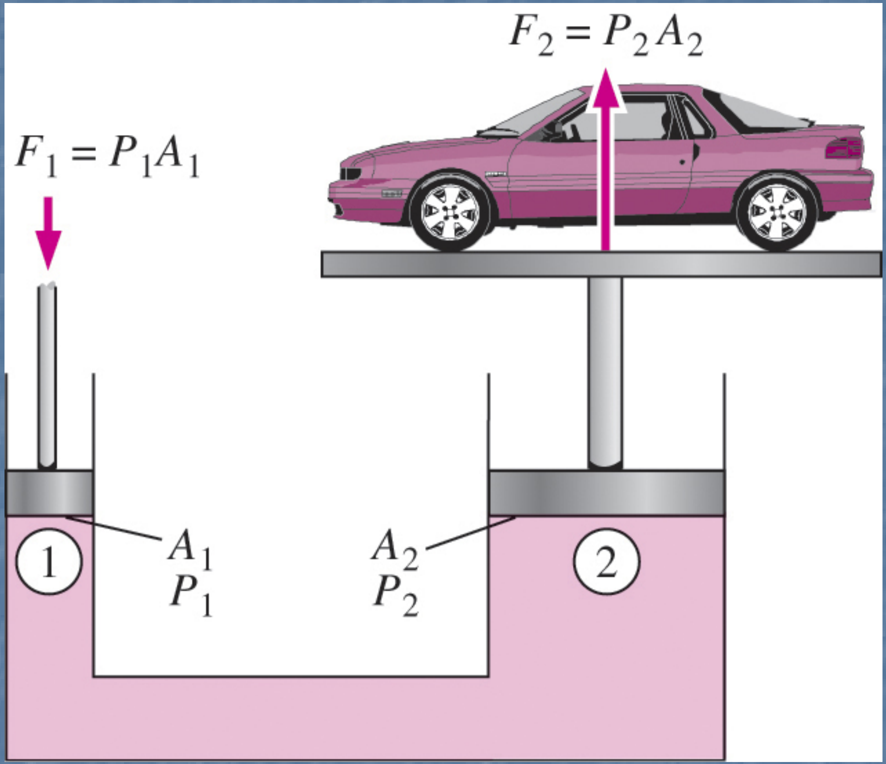
PRESSURE TRANSMISSION (PASCAL LAW)

In a closed system, a pressure produced at a point will be transmitted through out the entire system. (See Fig. 2)

Fluid Statics

Pascal's law: The pressure applied to a confined fluid increases the pressure throughout by the same amount.

$$P_1 = P_2 \quad \rightarrow \quad \frac{F_1}{A_1} = \frac{F_2}{A_2} \quad \rightarrow \quad \frac{F_2}{F_1} = \frac{A_2}{A_1}$$



Lifting of a large weight by a small force by the application of Pascal's law.

EXAMPLE 3.1 LOAD LIFTED BY A HYDRAULIC JACK

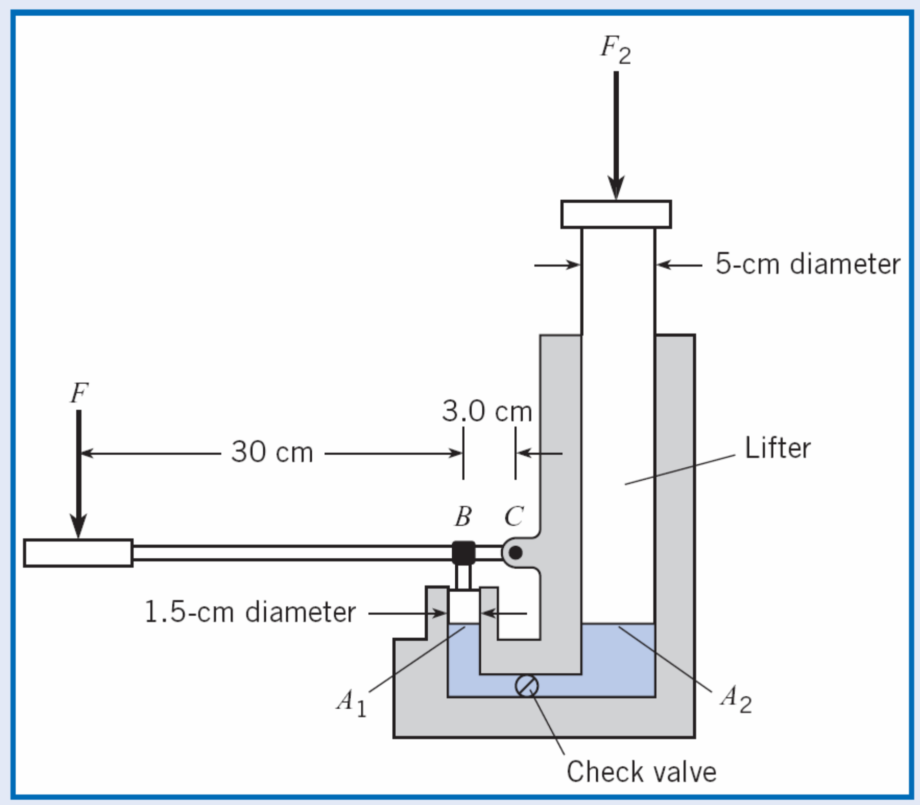
A hydraulic jack has the dimensions shown. If one exerts a force F of 100 N on the handle of the jack, what load, F_2, can the jack support? Neglect lifter weight.

Problem Definition

Situation: A force of $F = 100$ N is applied to the handle of a jack.

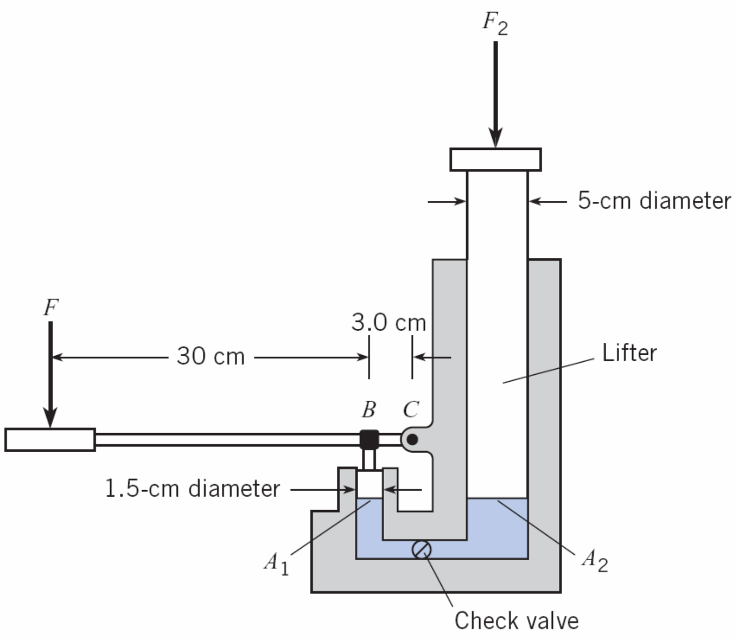
Find: Load F_2 in kN that the jack can lift.

Assumptions: Weight of the lifter component (see sketch) is negligible.



Plan

1. Calculate force acting on the small piston by applying moment equilibrium.
2. Calculate pressure p_1 in the hydraulic fluid by applying force equilibrium.
3. Calculate the load F_2 by applying force equilibrium.



Solution

1. Moment equilibrium

$$\sum M_C = 0$$

$$(0.33\text{ m}) \times (100\text{ N}) - (0.03\text{ m})F_1 = 0$$

$$F_1 = \frac{0.33\text{ m} \times 100\text{ N}}{0.03\text{ m}} = 1100\text{ N}$$

2. Force equilibrium (small piston)

$$\sum F_{\text{small piston}} = p_1 A_1 - F_1 = 0$$

$$p_1 A_1 = F_1 = 1100\text{ N}$$

Thus

$$p_1 = \frac{F_1}{A_1} = \frac{1100\text{ N}}{\pi d^2/4} = 6.22 \times 10^6\text{ N/m}^2$$

$$F_2 = p_1 A_2 = \left(6.22 \times 10^6\ \frac{\text{N}}{\text{m}^2}\right)\left(\frac{\pi}{4} \times (0.05\text{ m})^2\right) = \boxed{12.2\text{ kN}}$$

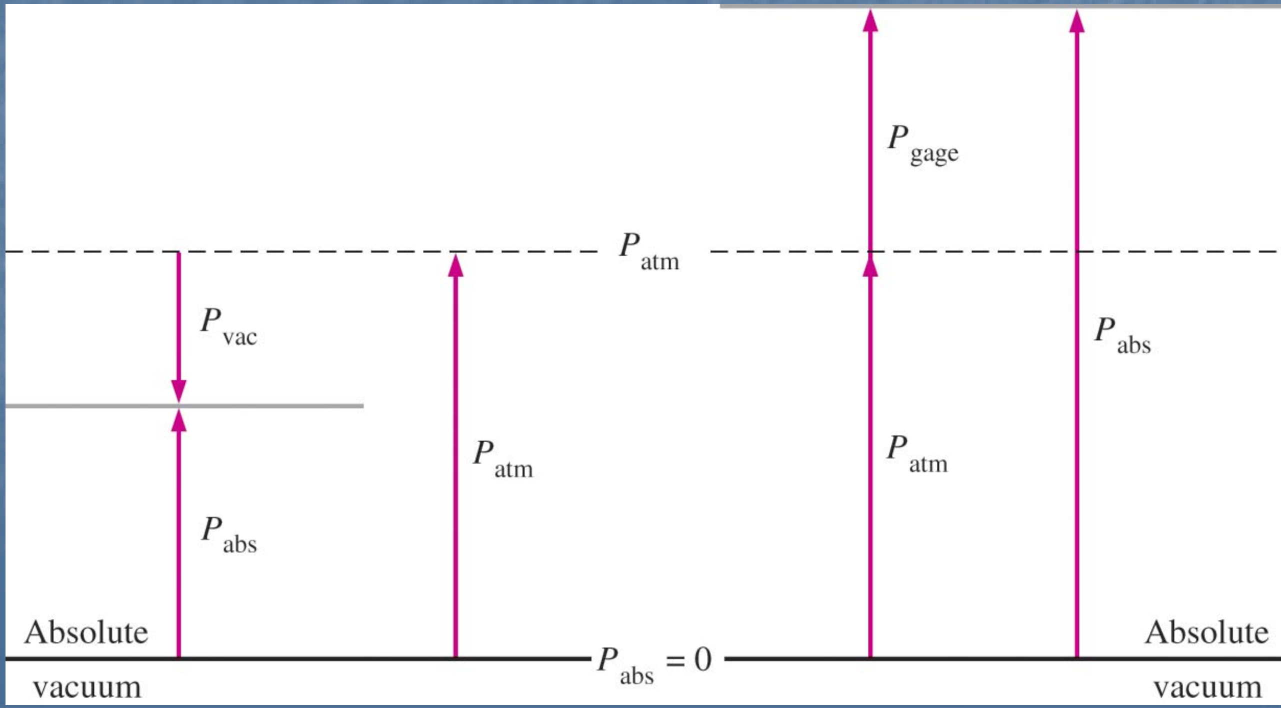
Fluid Statics

ABSOLUTE, GAUGE & VACUUM PRESSURE

- **Absolute pressure:** The actual pressure at a given position. It is measured relative to absolute vacuum (i.e., absolute zero pressure).
- **Gauge pressure:** The difference between the absolute pressure and the local atmospheric pressure. Most pressure-measuring devices are calibrated to read zero in the atmosphere, and so they indicate gage pressure.
- **Vacuum pressures:** Pressures below atmospheric pressure.

$$p_{(gauge)} + p_{(atm)} = p_{(absolute)}$$

$$P_{vac} = P_{atm} - P_{abs}$$



END OF LECTURE (1)